1.4 Motion along an Incline

SECTION EXPECTATIONS

- Analyze the motion of objects along inclined planes.
- Use vector and free-body diagrams to analyze forces.
- Predict and explain motion along inclined planes.

When you watch speed skiers, it appears as though there is no limit to the rate at which they can accelerate. In reality, their acceleration is always less that that of a free-falling object, because the skier is being accelerated by only a component of the force of gravity and not by the total force. Using the principles of dynamics and the forces affecting the motion, you can predict details of motion along an inclined plane.

Figure 1.17 Gravitational forces acting on downhill skiers have produced speeds greater than 241 km/h, even though only part of the total gravitational force accelerates a skier.

Choosing a Coordinate System for an Incline

The key to analyzing the dynamics and motion of objects on an inclined plane is choosing a coordinate system that simplifies the procedure. Since all of the motion is along the plane, it is convenient to place the *x*-axis of the coordinate system parallel to the plane, making the *y*-axis perpendicular to the plane, as shown in Figure 1.18.

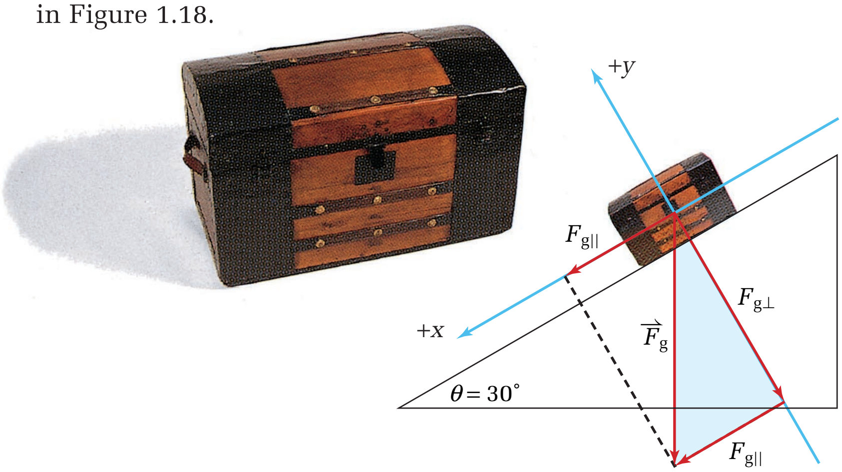


Figure 1.18 To find the components of the gravitational force vector, use the shaded triangle. Note that $\vec{F}_g$ is perpendicular to the horizontal line at the bottom and $F_{g\perp}$ is perpendicular to the plane of the ramp. Since the angles between two sets of perpendicular lines must be equal, the angle (θ) in the triangle is equal to the angle that the inclined plane makes with the horizontal.

The force of gravity affects motion on inclined planes, but the force vector is at an angle to the plane. Therefore, you must resolve the gravitational force vector into components parallel to and perpendicular to the plane, as shown in Figure 1.18. The component of force parallel to the plane influences the acceleration of the object and the perpendicular component affects the magnitude of the friction. Since several forces in addition to the gravitational force can affect the motion on an inclined plane, free-body diagrams are essential in solving problems, as shown in the sample problem below.

SAMPLE PROBLEM

Sliding Down an Inclined Plane

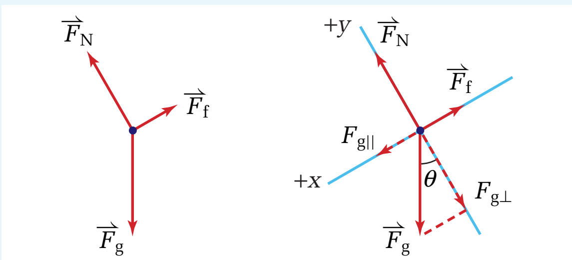
You are holding an 85 kg trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

(a) If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?

(b) If the trunk slides 1.3 m before reaching the bottom of the ramp, for what time interval did it slide?

Conceptualize the Problem

- To start framing the problem, draw a *free-body diagram*.
- Beside the free-body diagram, draw a coordinate system with the *x-axis parallel* to the ramp. On the coordinate system, draw the forces and *components of forces* acting on the trunk.



- Let the direction pointing *down* the slope be the *positive* direction.
- To find the *normal force* that is needed to determine the magnitude of the *frictional force*, apply *Newton's second law* to the forces or components of forces that are *perpendicular* to the ramp.
- The *acceleration perpendicular* to the ramp is *zero*.
- The *component of gravity parallel* to the trunk causes the trunk to *accelerate* down the ramp.
- *Friction* between the trunk and the ramp *opposes* the motion.
- If the *net force* along the ramp is *positive*, the trunk will *accelerate* down the ramp.
- To find the *acceleration* of the trunk down the ramp, apply *Newton's second law* to the forces or components of forces *parallel* to the ramp.
- Given the *acceleration* of the trunk, you can use the *kinematic equations* to find *other quantities* of motion.

Identify the Goal

(a) The acceleration, $a_{\parallel}$, of the trunk along the ramp

(b) The time interval, Δt, for the trunk to reach the end of the ramp

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Identify the Variables

Known		Implied	Unknown		
$m = 85\ \text{kg}$	$\theta = 35°$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_\text{g}$	$\vec{F}_\text{f}$	v_f
$\mu = 0.42$	$\Delta d = 1.3\ \text{m}$	$v_\text{i} = 0$	$F_{\text{g}\parallel}$	$\vec{F}_\text{N}$	$a_\parallel$
		$a_\perp = 0$	$F_{\text{g}\perp}$		

Develop a Strategy

Apply Newton's second law to the forces perpendicular to the ramp. Refer to the diagram to find all of the forces that are perpendicular to the ramp. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_\text{N} + F_{\text{g}\perp} = ma_\perp$$
$$F_\text{N} - mg\cos\theta = ma_\perp$$
$$F_\text{N} = mg\cos\theta + ma_\perp$$

Insert values and solve. Note that the acceleration perpendicular to the ramp ($a_\perp$) is zero.

$$F_\text{N} = (85\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 35° + 0$$
$$F_\text{N} = 683.05\ \text{N}$$

Apply Newton's second law to the forces parallel to the ramp. Refer to the diagram to find all of the forces that are parallel to the ramp. Solve for the acceleration parallel to the ramp.

$$\vec{F} = m\vec{a}$$
$$F_{\text{g}\parallel} + F_\text{f} = ma_\parallel$$
$$F_\text{f} = \mu F_\text{N} \text{ in negative direction}$$
$$mg\sin\theta - \mu F_\text{N} = ma_\parallel$$
$$a_\parallel = \frac{mg\sin\theta - \mu F_\text{N}}{m}$$

Insert values and solve.

$$a_\parallel = \frac{(85\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 35° - (0.42)(683.05\ \text{N})}{85\ \text{kg}}$$
$$a_\parallel = 2.251\ 71\ \frac{\text{m}}{\text{s}^2}$$
$$a_\parallel \cong 2.3\ \frac{\text{m}}{\text{s}^2}$$

(a) The acceleration of the trunk down the ramp is 2.3 m/s^2.

Apply the kinematic equation that relates displacement, acceleration, initial velocity, and time interval. Given that the initial velocity was zero, solve the equation for the time interval.

$$\Delta d = v_\text{i}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta t^2 = \frac{2\Delta d}{a}$$
$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Insert values and solve.

$$\Delta t = \sqrt{\frac{2(1.3\ \text{m})}{2.251\ 71\ \frac{\text{m}}{\text{s}^2}}}$$
$$\Delta t = 1.075\ \text{s}$$
$$\Delta t \cong 1.1\ \text{s}$$

(b) The trunk slid for 1.1 s before reaching the end of the ramp.

Validate the Solution

(a) Since the ramp is not at an extremely steep slope and since there is a significant amount of friction, you would expect that the acceleration would be much smaller than 9.81 m/s^2, which it is.

(b) The ramp is very short, so you would expect that it would not take long for the trunk to reach the bottom of the ramp. A time of 1.1 s is quite reasonable.